

12/4/17 "Those who look outside dream, those who look inside awaken." -Carl Jung

HW: Implicit Differentiation w/s #1, 2, 7, 21, 22  
Test 2 on Thursday 12/14

AIM:

How do we write equations of tangent lines to curves using Implicit Diff?

Warm Up:

- 1) Find an equation for the line tangent to
- $\frac{x}{x+2y} = y$
- 
- in the 4th Quadrant when
- $x=1$

Point:

$$\frac{1}{1+2y} = \frac{y}{1}$$

$$1 = y + 2y^2$$

$$0 = 2y^2 + y - 1$$

$$a = -2$$

$$b = 1$$

$$0 = 2y^2 + 2y - 1y - 1$$

$$0 = 2y(y+1) - 1(y+1)$$

$$0 = (2y-1)(y+1)$$

$$y = \frac{1}{2} \quad y = -1$$

Point:

$$(1, -1)$$

$$\text{Points } (1, \frac{1}{2}) \text{ and } (1, -1)$$

1st Quad

4th Quad

Slope:

$$\frac{d}{dx} \left( \frac{x}{x+2y} = y \right) = \frac{(x+2y)(1) - (1+2\frac{dy}{dx})(x)}{(x+2y)^2} = \frac{dy}{dx}$$

⊗ We know the point is  $(1, -1)$  so we can plug it in to find the slope.

$$\frac{(1+2(-1))(1) - (1+2\frac{dy}{dx})(1)}{(1+2(-1))^2} = \frac{dy}{dx}$$

$$\frac{-1 - 1 - 2\frac{dy}{dx}}{(-1)^2} = \frac{dy}{dx}$$

$$\frac{-2 - 2\frac{dy}{dx}}{+2\frac{dy}{dx} + 2\frac{dy}{dx}} = \frac{dy}{dx}$$

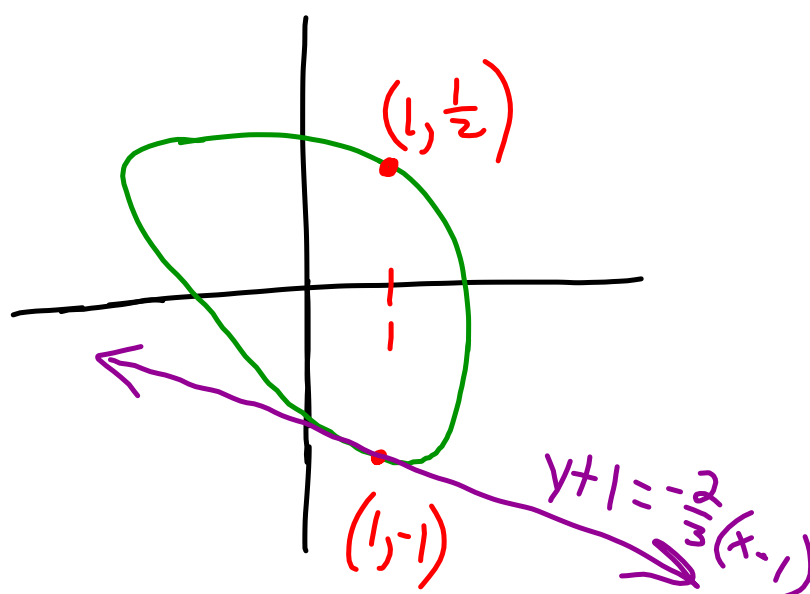
$$\frac{-2}{3} = \frac{3\frac{dy}{dx}}{3}$$

$$\frac{dy}{dx} = -\frac{2}{3} = \text{slope}$$

Point:

$$(1, -1)$$

$$y - (-1) = -\frac{2}{3}(x - 1)$$



$$21) \quad x^2 + y^2 \textcircled{-xy} + 3x - 9 = 0$$

$$2x + 2y \frac{dy}{dx} - 1y + -x \frac{dy}{dx} + 3 + 0 = 0$$

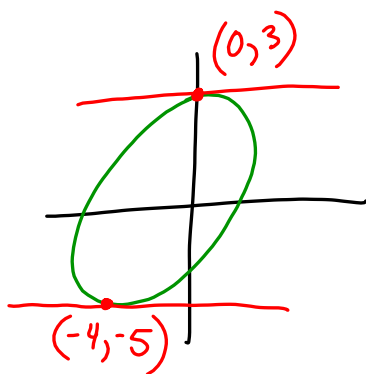
$$-2y \frac{dy}{dx}$$

$$+ x \frac{dy}{dx}$$

$$-2y \frac{dy}{dx} + x \frac{dy}{dx}$$

$$\frac{2x - y + 3}{-2y + x} = \frac{\frac{dy}{dx} (-2y + x)}{-2y + x} \quad \leftarrow \text{Factor out } \frac{dy}{dx}$$

$$\textcircled{\frac{dy}{dx} = \frac{2x - y + 3}{-2y + x}}$$



22) Horizontal tangents when slope = 0

$$\frac{dy}{dx} = \frac{2x - y + 3}{-2y + x}$$

⊗ Numerator = 0

$$\frac{dy}{dx} = 0$$

$$\begin{array}{r} 2x - y + 3 = 0 \\ + y \quad + y \\ \hline \end{array}$$

$$\underline{2x + 3 = y}$$

substitute  
into original

$$x^2 + (2x + 3)^2 - x(2x + 3) + 3x - 9 = 0$$

$$x^2 + 4x^2 + 12x + 9 - 2x^2 - 3x + 3x - 9 = 0$$

$$3x^2 + 12x = 0$$

$$3x(x + 4) = 0$$

$$x = 0 \quad x = -4$$

Find y:

$$0^2 + y^2 - 0y + 3(0) - 9 = 0$$

$$y^2 - 9 = 0$$

$$(y + 3)(y - 3) = 0$$

$$y = -3 \quad y = 3$$

reject

$$\frac{dy}{dx} \neq 0$$

$$\textcircled{0, 3}$$

$$\begin{array}{l} (0, 3) \\ (-4, -5) \end{array}$$

$$(-4)^2 + y^2 - (-4)y + 3(-4) - 9 = 0$$

$$16 + y^2 + 4y - 12 - 9 = 0$$

$$y^2 + 4y - 5 = 0$$

$$(y + 5)(y - 1) = 0$$

$$y = -5 \quad y = 1$$

reject

$$\frac{dy}{dx} \neq 0 \textcircled{-4, 1}$$